

Another coincidence problem for Λ CDM?

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Over the last nine years of cosmic microwave background observations, the Wilkinson Microwave Anisotropy Probe (*WMAP*) results were consistent with a Λ CDM cosmological model in which the age of the Universe is one Hubble time, and the time-averaged value of the deceleration parameter is consistent with zero. This curious observation has been put forward as a new coincidence problem for the Λ CDM concordance cosmology, which is in fact a ‘greater’ coincidence than the near equality of the density parameters of matter and the cosmological constant. At the moment of writing these conference proceedings, the Planck Collaboration has released its first cosmological data, which revealed a small shift in the Λ CDM cosmological parameters when compared to *WMAP*. We show that under the assumption of a spatially flat Λ CDM cosmology, Planck’s results remove this coincidence problem for Λ CDM at greater than 99% confidence level.

Komatsu et al.¹ presented the combination of seven-year data from *WMAP* and improved astrophysical data, yielding constraints on the basic six parameters of a flat Λ CDM model. Their maximum likelihood parameter for Ω_{Λ_o} is 0.728, and $\Omega_{\Lambda_o} = 0.725 \pm 0.016$ is their mean and 1σ of the posterior distribution^a. We now show that precisely this value of Ω_{Λ_o} is related to a present day age of the Universe that is consistent with one Hubble time if we assume a spatially flat Λ CDM cosmology, and that the coincidence is significantly weakened in the Planck data.

Starting from Einsteins equations of General Relativity and the Friedmann-Lemaître-Robertson-Walker metric, one can write the Friedmann equation for a spatially flat Universe as

$$\frac{da}{dt} = H_o \sqrt{a^{-1}\Omega_{m_o} + a^2\Omega_{\Lambda_o}}. \quad (1)$$

with a the scale factor, Ω_{m_o} is the present day normalised matter density, Ω_{Λ_o} is the corresponding energy density of dark energy, and H_o the present day value of the Hubble parameter. Note that the equation of state of dark energy required for the expansion given by Equation 1 is $w = -1$, equivalent to a standard cosmological constant. Furthermore, the energy density in radiation has been neglected, as it was only dominant in the early stages of the Universe. Integration of this equation

^aSee the columns ‘*WMAP*+BAO+ H_o ML’ and ‘*WMAP*+BAO+ H_o Mean’ in their Table 1. Throughout this text, a subscript o denotes the present day value.

results in an expression for the age of the Universe in terms of a :

$$H_o t = \frac{2}{3\sqrt{\Omega_{\Lambda_o}}} \cosh^{-1} \left(\sqrt{1 + \frac{a^3 \Omega_{\Lambda_o}}{\Omega_{m_o}}} \right). \quad (2)$$

Today, $t = t_o$ and $a_o \equiv 1$, which means that $H_o t_o = 1$, if and only if

$$e^{-3\sqrt{\Omega_{\Lambda_o}}} (\Omega_{\Lambda_o} - 1) + e^{3\sqrt{\Omega_{\Lambda_o}}} (\Omega_{\Lambda_o} - 1) + 2\Omega_{\Lambda_o} + 2 = 0 \quad (3)$$

where, through flatness, it is assumed that the density parameter of matter $\Omega_m = 1 - \Omega_{\Lambda}$. Because equation 2 requires $\Omega_{\Lambda_o} > 0$, the only solution to equation 3 is $\Omega_{\Lambda_o} \approx 0.737125$. This special value for Ω_{Λ_o} laid within the 1σ of the posterior distribution after seven-year *WMAP*+BAO+ H_o observations quoted above,¹ however in the nine-year *WMAP*+BAO+ H_o cosmology results $\Omega_{\Lambda_o} = 0.712 \pm 0.010$.² In general, we can write the time-averaged deceleration parameter $\langle q \rangle$ as a function of a :

$$\langle q \rangle + 1 = \frac{1}{tH} = \left[\frac{2}{3} \sqrt{\left(\frac{\Omega_{m_o}}{a^3 \Omega_{\Lambda_o}} + 1 \right)} \cosh^{-1} \left(\sqrt{1 + \frac{a^3 \Omega_{\Lambda_o}}{\Omega_{m_o}}} \right) \right]^{-1} \quad (4)$$

In the top right panel of Figure 1, we plot $\langle q \rangle$ using the 95% confidence levels of Ω_{Λ_o} from both the Planck data³ ($0.686^{+0.037}_{-0.040}$) and the WMAP 9-year data² (0.721 ± 0.050) without BAO+ H_o priors. The probability density function of the posterior distribution is indicated by the colorscale. Note that $\langle q \rangle_o$ is close to zero only during a brief period in cosmic time, and with the small errors on the derived value of Ω_{Λ_o} it is remarkable that $\langle q \rangle_o \approx 0$ in the WMAP 9-year data.² However, $\langle q \rangle_o \neq 0$ for the Planck data,³ which is clearest in the zoom-in in the top right panel of Figure 1 near a_o . This is an updated version of the top left panel in Figure 1, in which the Komatsu et al. *WMAP*+BAO+ H_o data¹ is used.

The coincidence that $\langle q \rangle_o \approx 0$ was first discovered by Lima.⁴ Figure 1 is an adapted version of Figure 1 in van Oirschot et al.,⁵ who also showed that $\langle q \rangle_o \approx 0$ is the same as the $R_h \approx ct$ coincidence reported by Melia et al.⁶ Comparing this coincidence with the well known coincidence problem for Λ CDM, the near equality of the density parameters of matter and the cosmological constant, we see that this new coincidence is in fact a ‘greater’ one (see the bottom panels of Figure 1). Whereas the density parameters were equal already a few Gyrs ago, $\langle q \rangle = 0$ happens instantaneously, where the red and blue solid line (bottom left panel) or red and green line (bottom right panel) cross.

Recently, Melia et al. proposed the “ $R_h = ct$ ” universe to naturally explain the coincidence and touted that it is superior to Λ CDM in explaining our observations of the Universe.^{7–9} However, see Bilicki and Seikel¹⁰ and Lewis¹¹ for arguments against the $R_h = ct$ universe. The Planck Collaboration³ finds $\Omega_{\Lambda_o} = 0.686^{+0.048}_{-0.052}$ with 99% confidence, which indicates that we have to go beyond the 99% confidence level to arrive at $R_h = ct$. Using the mean and 1σ of the Ω_{Λ_o} posterior distribution from Planck gives $R_{h_o} = 1.05 \pm 0.02 \text{ } ct_o$, or $\langle q \rangle_o = 0.05 \pm 0.02$. Therefore, we can exclude the proposed $R_h = ct$ universe on the basis of the Planck data and the new coincidence problem for Λ CDM is significantly weakened.

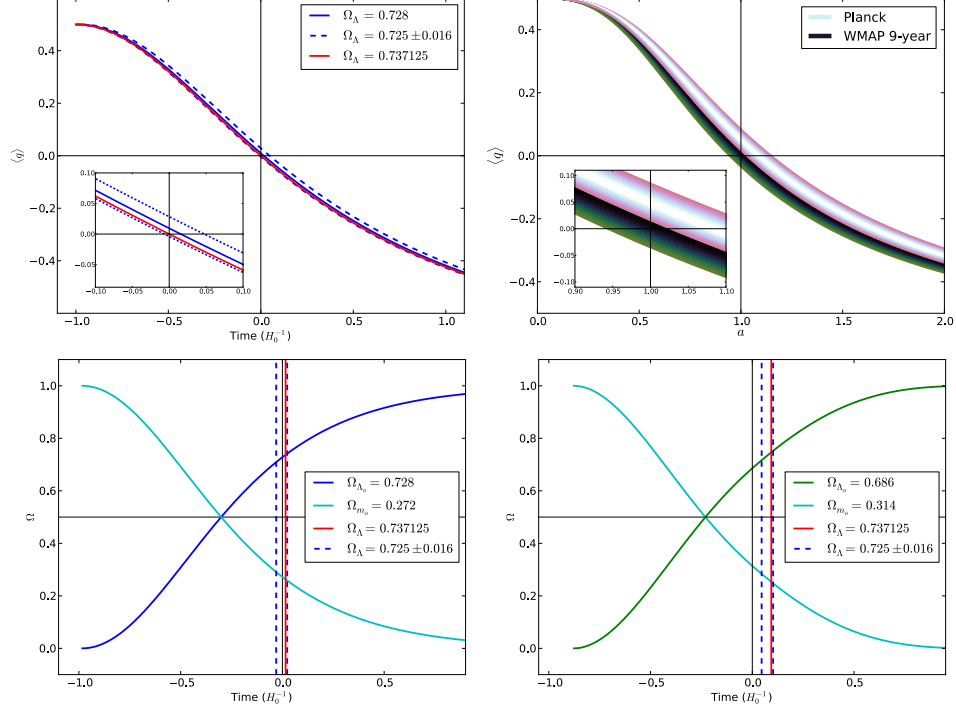


Fig. 1. Top left: The blue dashed and solid lines correspond to the 68% confidence levels on the mean value of the posterior distribution and the maximum likelihood of Ω_{Λ_o} from Komatsu et al.¹ The red line shows the cosmological model in which $\langle q \rangle_o = 0$, thus $\Omega_{\Lambda} = 0.737125$. Top right: An updated version of the top left panel, comparing $\langle q \rangle$ as a function of the cosmological scale factor, $a(t)$, using the 95% confidence levels of Ω_{Λ_o} in the WMAP 9-year and Planck data. The brighter (darker) the colors, the more confident the Planck (WMAP 9-year) data. Bottom panels: The Komatsu et al.¹ maximum likelihood of Ω_{Λ_o} is used for the blue solid line in the left panel, that from the Planck Collaboration for the green line in the right panel. The cyan lines indicate the evolution of the corresponding density parameter of matter. The coincidence is apparent in the left panel, because t_0 lays between the dashed lines (~ 743 Myr) that indicate the 1σ of the posterior distribution of Ω_{Λ_o} from Komatsu et al.,¹ but it has disappeared in the right panel.

References

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